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INTERACTION OF AN EXPLOSION BUBBLE WITH A FIXED RIGID STRUCTURE

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SUMMARY

In the recent times, the boundary integral method has been utilised extensively in the study of bubble dynamics. This paper presents a modified form of the boundary integral method to model the motion of a bubble close to a fixed rigid structure. The resulting integral equations are solved using the boundary element method, and the system is integrated through time using a simple Euler scheme. Finally, the results of the model are presented to predict the motion of a bubble in a number of typical axisymmetric situations. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: explosion; bubble; boundary elements

1. INTRODUCTION

The behaviour of underwater explosions has been systematically investigated in the recent times, beginning with mainly experimental investigations undertaken during the second world war. Recently there has been considerable interest in the problem of determining the motion of an explosion bubble, especially when there is a submerged structure close to the point of the explosion. Typically, the initial radius of such bubbles is relatively small and the high pressure of the gas inside the bubble causes it to expand. Eventually the hydrostatic pressure of the surrounding water will halt the growth of the bubble and cause it to collapse, until the internal pressure causes it to rebound and start growing again. However, it has been observed that as the bubble collapses it forms a re-entrant jet, the direction of which is dependent on the geometry of the fluid region in which the bubble exists. Often the direction of this jet is of interest since it maybe directed towards a nearby structure immersed in the water, and it is possible that the impact of the jet is a mechanism for causing damage to the structure.

A number of mathematical models have been proposed for solving the problem of determining the bubble's motion. One such model is based on the boundary integral method. An axisymmetric formulation has been used by Blake *et al.* [1,2] to study the motion of bubbles close to infinite rigid boundaries or free-surfaces. Best [3] proposed a modification to the axisymmetric boundary integral method, to model the motion of the toroidal bubble that results from the jet penetrating the opposite side of the bubble. Other models, such as those of Chahine [4] and Harris [5,6], have employed a fully three-dimensional boundary integral method to predict the motion of a bubble close to a rigid structure in the water. Clearly,

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models such as these can be used to determine the complete motion of the bubble and surrounding fluid, and hence the direction of the bubble's jet can be found directly from the results of the simulation.

This paper shows how the boundary integral method, with an axisymmetric formulation, can be used to model the motion of a bubble close to a fixed rigid structure, such as a sphere, cylinder or plate, immersed in an infinite fluid. Section 2 introduces a simple mathematical model that can be used to study the motion of the bubble in a fluid. In Section 3, the problem is reformulated as an integral equation over the surfaces of the bubble and rigid structure, and a numerical scheme for obtaining its solution is presented. Section 4 presents the results for the motion of a bubble close to a number of different rigid structures, under the influence of gravity.

2. MATHEMATICAL MODEL

This section presents a suitable model for determining the motion of a bubble close to a fixed rigid structure by making the standard assumptions [1,2] that the fluid is inviscid, incompressible and irrotational. Therefore, the flow field can be described by a velocity potential ϕ, which is a solution of Laplace's equation [7]:

$$\nabla^2\phi = 0. \tag{1}$$

The contents of the bubble (if any) are assumed to be ideal and the thermodynamic processes are assumed to be polytropic with constant γ. Hence, the pressure, P_b, within the bubble is given by

$$P_\mathrm{b} = P_0\left(\frac{V_0}{V(t)}\right)^{\gamma}, \tag{2}$$

where V_0 is the initial volume of the bubble, $V(t)$ is the volume of the bubble at some later time t, and P_0 is the initial pressure inside the bubble. If P_∞ denotes the far-field pressure in the $z = 0$ plane, it is possible to write the Bernoulli equation for any point in the fluid as [7]:

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \frac{P}{\rho} + gz = \frac{P_\infty}{\rho}, \tag{3}$$

where ρ is the density of the fluid, g is the acceleration due to gravity assumed to be directed parallel to the negative z-axis, and P denotes the pressure at the point in the fluid. Since the fluid pressure at the surface of the bubble must be equal to the pressure of the gas inside the bubble, Bernoulli's equation yields

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \frac{P_0}{\rho}\left(\frac{V_0}{V(t)}\right)^{\gamma} + gz = \frac{P_\infty}{\rho}, \tag{4}$$

at all points on the surface of the bubble. There cannot be a fluid flow perpendicular to the surface of a fixed rigid structure, thus $\partial\phi/\partial n = 0$ on all such surfaces. The initial conditions of the system are that the initial potential on the bubble surface and the initial internal pressure of the bubble are known. For a cavitation bubble, the initial potential is taken as the Rayleigh solution [8], and the internal pressure is assumed to be zero; whilst for an explosion bubble, the initial potential is zero but there is a large excess pressure inside the bubble.

3. NUMERICAL ANALYSIS

In this formulation, the fluid domain (Ω) is assumed to be unbounded, and so there are obvious problems with the use of domain-based numerical methods, such as the finite element method, to solve the underlying differential equation. For this reason, the boundary integral method has proved popular for solving problems such as this, as the three-dimensional infinite domain differential equation problem is transformed into a two-dimensional integral equation defined on a finite region, namely the surfaces of the bubble and the rigid structure. It can be shown that the velocity potential ϕ and the normal derivative $\partial\phi/\partial n$ must satisfy Green's second theorem [9]:

$$\int_S \left\{\phi(q)\frac{\partial G}{\partial n}(p,q) - G(p,q)\frac{\partial \phi}{\partial n}(q)\right\} \mathrm{d}S_q = \begin{cases} \phi(p) & \text{if } p\in\Omega \\ \frac{1}{2}\phi(p) & \text{if } p\in S \\ 0 & \text{otherwise} \end{cases}, \tag{5}$$

where S denotes the union of the surface of the bubble and the surface of the rigid structure and $G(p, q)$ is the free-space Green's function:

$$G(p,q) = \frac{1}{4\pi|p-q|}. \tag{6}$$

For $p\in S$, Equation (5) yields a first kind Fredholm integral equation for $\partial\phi/\partial n$ if ϕ is known, and a second kind Fredholm integral equation for ϕ if $\partial\phi/\partial n$ is known. If $p\in S_{\mathrm{b}}$ or $p\in S_{\mathrm{r}}$, where the subscripts b and r denote the quantities on the surface of the bubble and the quantities on the rigid structure respectively, the integrals appearing in (5) can be split to yield:

$$\frac{1}{2}\phi_{\mathrm{b}} = \int_{S_{\mathrm{b}}}\left(\phi_{\mathrm{b}}\frac{\partial G}{\partial n} - G\frac{\partial \phi_{\mathrm{b}}}{\partial n}\right)\mathrm{d}S + \int_{S_{\mathrm{r}}}\left(\phi_{\mathrm{r}}\frac{\partial G}{\partial n} - G\frac{\partial \phi_{\mathrm{r}}}{\partial n}\right)\mathrm{d}S \quad p\in S_{\mathrm{b}}, \tag{7}$$

$$\frac{1}{2}\phi_{\mathrm{r}} = \int_{S_{\mathrm{b}}}\left(\phi_{\mathrm{b}}\frac{\partial G}{\partial n} - G\frac{\partial \phi_{\mathrm{b}}}{\partial n}\right)\mathrm{d}S + \int_{S_{\mathrm{r}}}\left(\phi_{\mathrm{r}}\frac{\partial G}{\partial n} - G\frac{\partial \phi_{\mathrm{r}}}{\partial n}\right)\mathrm{d}S \quad p\in S_{\mathrm{r}}. \tag{8}$$

This analysis can be generalised to more than two surfaces. The effects of an infinite rigid plane on the bubble motion can be included by using the modified Green function

$$G(p,q) = \left\{\frac{1}{4\pi|p-q|} + \frac{1}{4\pi|p'-q|}\right\}, \tag{9}$$

where p' is the image of p in the rigid boundary, as used by Blake *et al.* [1]. To further simplify the problem, it is assumed that the fluid domain and flow are axisymmetric about the z-axis. In this situation, it is convenient to use cylindrical polar co-ordinates (r, θ, z), and note that due to axial symmetry, neither ϕ nor $\partial\phi/\partial n$ will depend on the angle θ, thus reducing the size of the computational problem. The surface of the bubble and the structure are generated by rotating some appropriate curve about the z-axis. For both surfaces, the generating curve in the (r, z) plane can be represented parametrically by two functions $r(s)$ and $z(s)$, where s is some appropriate parameter, such as the arc-length. The surface S_{b} of the bubble is discretised by choosing $n+1$ node points $(r_0, z_0), \ldots, (r_n, z_n)$ with $z_0 \le z_1 \le \cdots \le z_n$, along the generating curve. The parametric functions $r(s)$ and $z(s)$ are interpolated using clamped cubic splines [10], where the endpoint clamped conditions are $\mathrm{d}r/\mathrm{d}s = +1$ and $\mathrm{d}z/\mathrm{d}s = 0$ at z_0 and $\mathrm{d}r/\mathrm{d}s = -1$ and $\mathrm{d}z/\mathrm{d}s = 0$ at z_n. Similarly, as the value of the velocity potential ϕ_{b} is known at the node points, it can be interpolated by a clamped cubic spline with $\mathrm{d}\phi/\mathrm{d}s = 0$ at both ends. The unknown normal derivative of the potential, ψ, is approximated by using the piecewise linear interpolation scheme:

$$\psi_{\mathrm{b}}(s)=\psi_{\mathrm{b}_{j-1}}\left[\frac{(s_{j-1}-s)}{s_j-s_{j-1}}\right]+\psi_{\mathrm{b}_j}\left[\frac{(s-s_j)}{s_j-s_{j-1}}\right] \quad s_{j-1}\leq s\leq s_j, \tag{10}$$

where s_j and ψ_{b_j} are the respective values of the parameter s and the normal derivative of the potential at the jth node.

The surface of the rigid structure is discretised at m node points, but in this case the generating curves $r(s)$ and $z(s)$ and the potential ϕ are interpolated using a simple piecewise linear scheme, as is the normal derivative of the potential, ψ_{r}. Further details of these approximations can be found in Amini *et al.* [11].

Using the approximate surfaces, potentials and the normal derivative of the potential described above, it is possible to discretise Equations (7) and (8) using the collocation method [12] to obtain a block matrix equation of the form

$$\begin{pmatrix} M_{\mathrm{bb}} & M_{\mathrm{br}} \\ M_{\mathrm{rb}} & M_{\mathrm{rr}} \end{pmatrix}\begin{pmatrix} \underline{\phi_{\mathrm{b}}} \\ \underline{\phi_{\mathrm{r}}} \end{pmatrix}=\begin{pmatrix} L_{\mathrm{bb}} & L_{\mathrm{br}} \\ L_{\mathrm{rb}} & L_{\mathrm{rr}} \end{pmatrix}\begin{pmatrix} \underline{\psi_{\mathrm{b}}} \\ \underline{\psi_{\mathrm{r}}} \end{pmatrix}, \tag{11}$$

where $\underline{\phi_{\mathrm{b}}}$ is the vector of nodal values of ϕ_{b} and $\underline{\phi_{\mathrm{r}}}$, $\underline{\psi_{\mathrm{b}}}$, $\underline{\psi_{\mathrm{r}}}$ are similarly defined. Since the unknown quantities are the normal velocity on S_{b} and the potential on S_{r}, Equation (11) can be rearranged to obtain

$$\begin{pmatrix} L_{\mathrm{bb}} & -M_{\mathrm{br}} \\ L_{\mathrm{rb}} & -M_{\mathrm{rr}} \end{pmatrix}\begin{pmatrix} \underline{\psi_{\mathrm{b}}} \\ \underline{\phi_{\mathrm{r}}} \end{pmatrix}=\begin{pmatrix} M_{\mathrm{bb}} & -L_{\mathrm{br}} \\ M_{\mathrm{rb}} & -L_{\mathrm{rr}} \end{pmatrix}\begin{pmatrix} \underline{\phi_{\mathrm{b}}} \\ \underline{\psi_{\mathrm{r}}} \end{pmatrix}, \tag{12}$$

which can be solved for $\underline{\psi_{\mathrm{b}}}$ and $\underline{\phi_{\mathrm{r}}}$. Once both the potential and its normal derivative are known on the surface of the bubble, it is possible to compute the components of the fluid velocity $\partial\phi/\partial r$ and $\partial\phi/\partial z$ at each node by solving

$$\frac{\partial\phi}{\partial n}=\frac{\partial\phi}{\partial r}n_r+\frac{\partial\phi}{\partial z}n_z, \tag{13}$$

$$\frac{\partial\phi}{\partial s}=\frac{\partial\phi}{\partial r}\frac{\partial r}{\partial s}+\frac{\partial\phi}{\partial z}\frac{\partial z}{\partial s}, \tag{14}$$

where $\partial\phi/\partial s$, $\partial r/\partial s$ and $\partial z/\partial s$ are obtained by differentiating the appropriate interpolating cubic splines and n_r and n_z denote the r and z components of the unit normal respectively.

The location of the bubble surface can now be updated using the simple Euler scheme

$$r_i(t+\delta t)=r_i(t)+\delta t\frac{\partial\phi_i}{\partial r}+O(\delta t^2), \tag{15}$$

$$z_i(t+\delta t)=z_i(t)+\delta t\frac{\partial\phi_i}{\partial z}+O(\delta t^2), \tag{16}$$

$$\phi_i(t+\delta t)=\phi_i(t)+\delta t\frac{\mathrm{D}\phi_i}{\mathrm{D}t}+O(\delta t^2), \tag{17}$$

where $\mathrm{D}\phi_i/\mathrm{D}t$ is the total or the material derivative given by

$$\frac{\mathrm{D}\phi_i}{\mathrm{D}t}=\frac{\partial\phi_i}{\partial t}+\left(\frac{\partial\phi_i}{\partial r}\right)^2+\left(\frac{\partial\phi_i}{\partial z}\right)^2 \tag{18}$$

and $\partial\phi_i/\partial t$ is computed from Bernoulli's equation (4).

The time step δt is chosen such that

$$\delta t = \frac{\delta \phi}{\max_i \left(\left| \frac{D\phi_i}{Dt} \right| \right)}, \tag{19}$$

where $\delta\phi$ is the maximum change allowed in the potential ϕ_b at any node. This is to try and ensure that the time stepping is stable. However, it is possible that $D\phi_i/Dt$ is very close to zero at every node, which would result in a very large time step. To avoid this, the additional constraint that δt can be at most k times the pervious time step, for some constant k, is employed.

4. COMPUTATIONAL RESULTS AND CONCLUSIONS

4.1. Non-dimensionalising the solution

In developing the numerical solution of the equations described in the previous sections, it is convenient to scale all terms, thus producing dimensionless equations. All lengths are scaled with respect to the maximum bubble radius, R_m, yielding the following dimensionless quantities,

$$Z = \frac{z}{R_m}, \qquad R = \frac{r}{R_m}.$$

Quantities involving time are either explicitly or implicitly scaled as

$$R_m \left(\frac{\rho}{P_\infty - P_b} \right)^{1/2};$$

whilst the pressure, potentials and buoyancy are scaled as

$$P = \frac{p - p_b}{p_0 - p_b}, \qquad \Phi = \frac{\phi}{R_m} \frac{\rho}{p_0 - p_b}, \qquad \delta = \left(\frac{\rho g R_m}{P_\infty - P_b} \right)^{1/2}.$$

All calculations are made in terms of the above non-dimensional variables.

Figure 1 shows the location of the bubble centroid for a bubble close to a fixed rigid disk of a large radius (labelled plate in the figure) and compare the location with that for a bubble close to an infinite rigid boundary (labelled boundary in the figure). Figure 2 gives the corresponding results for the bubble volume. Clearly the results from the two situations are in agreement, showing that this method yields results that are consistent with previous work [1].

Figure 3(a) and (b) show the growth and collapse respectively, of a bubble that is 1.0 unit above a fixed rigid cylinder. The radius of the cylinder is 1.0 unit and its height is 2.5 units. The entire configuration is in the buoyancy field with a buoyancy parameter, $\delta = 0.1$. The figures illustrates that if the bubble is close to the rigid structure then it jets downwards towards the rigid structure; whereas it would jet upwards due to buoyancy if the structure was not there.

Figure 4 shows the displacement of the centroid of the bubble at different initial distances above a fixed rigid sphere, in a mild buoyancy field (with the buoyancy parameter $\delta = 0.1$). The radius of the sphere is 3.5 units and the bubble is placed 1.0, 2.0, 3.0, 4.0 units respectively, above the fixed rigid sphere. This shows that whether or not the bubble ultimately moves towards or away from the structure depends on the initial distance of the bubble from

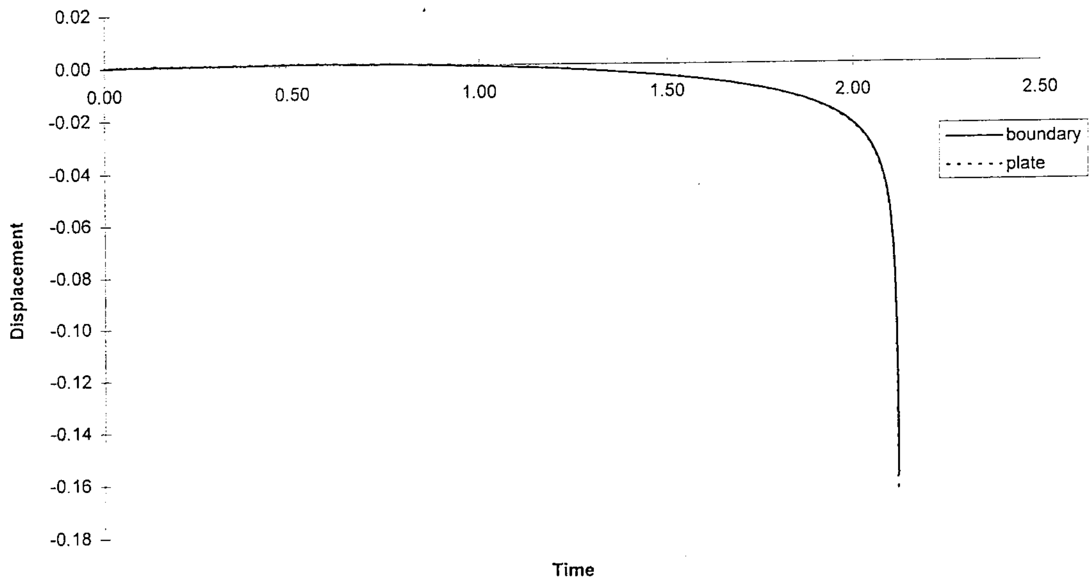

Figure 1. Comparison of the displacement of the centroid of the bubble above a fixed rigid boundary and a fixed rigid plate.

the structure. This effect is similar to that observed by Blake *et al.* [1] for a bubble near an infinite rigid boundary.

The boundary element method has been shown to be a useful tool in predicting the motion of a bubble in an infinite or semi-infinite potential flow situation. This paper has shown how

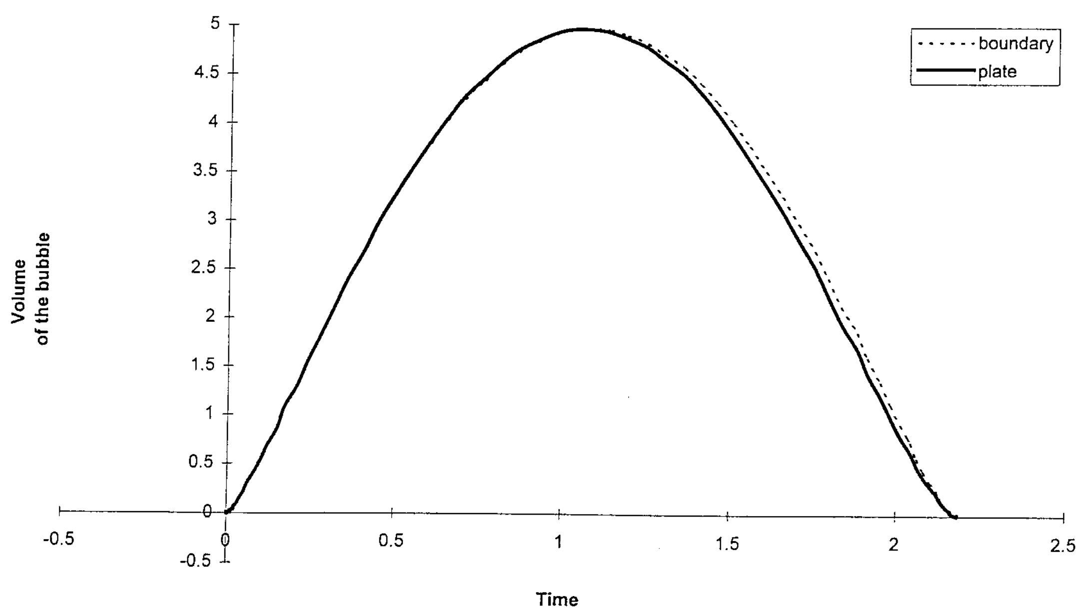

Figure 2. Comparison of the volume of the bubble above an infinite rigid boundary and a fixed rigid plate of a large radius.

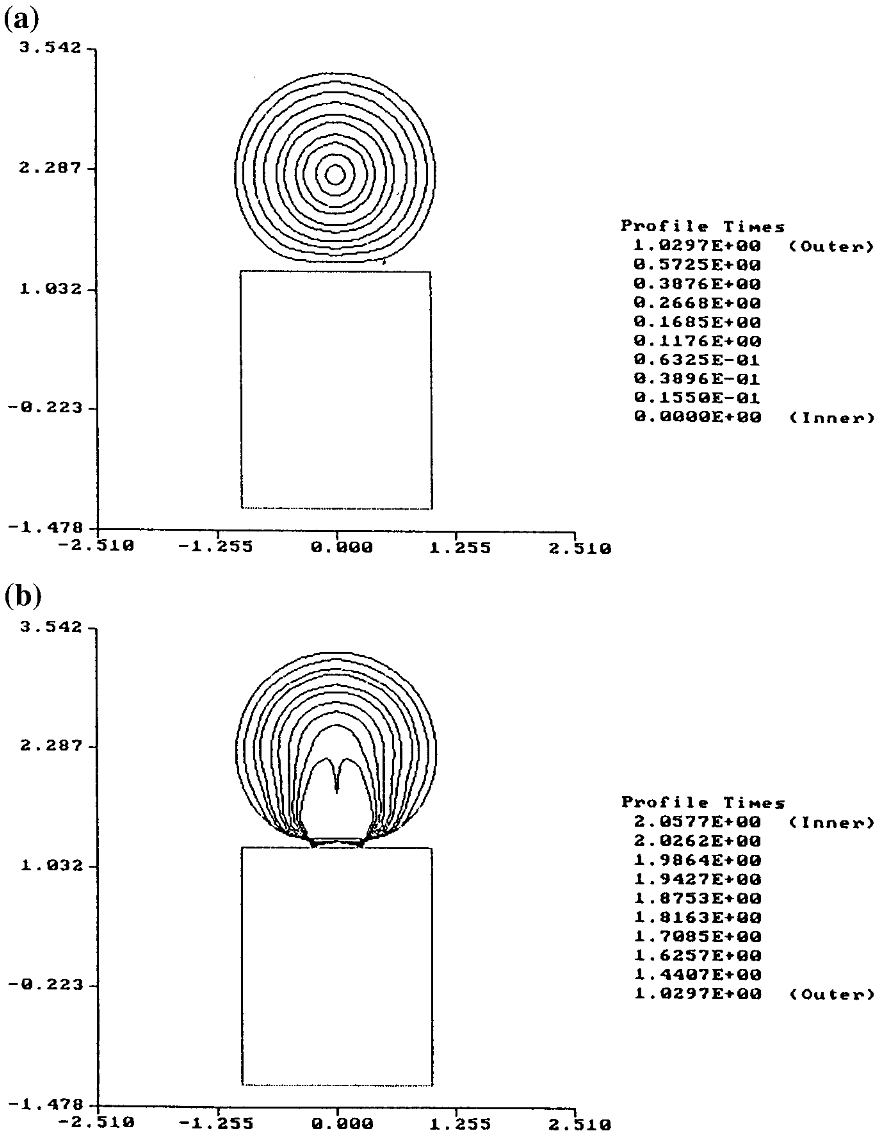

Figure 3. (a) Growth of an explosion bubble above a fixed rigid cylinder. (b) Collapse of an explosion bubble above a fixed rigid cylinder.

the basic method can be modified to include finite rigid structures immersed in the fluid close to the bubble. For relatively large rigid structures, the bubble essentially behaves like a bubble close to an infinite rigid boundary, whereas for structures that are approximately the same size as that of the bubble, the interaction between the bubble and the structure is more complicated.

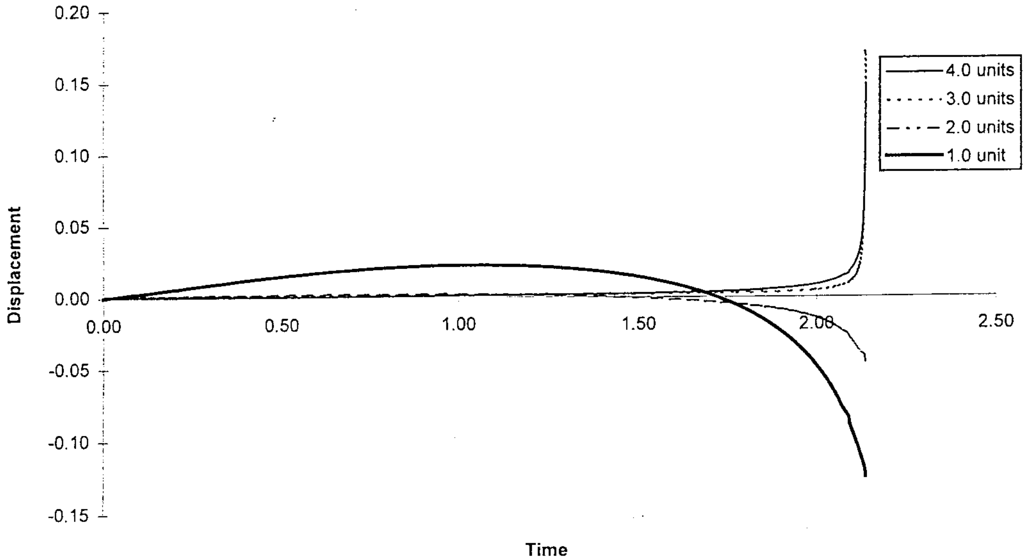

Figure 4. Displacement of the centroid of the explosion bubble at various distances above a fixed rigid sphere of radius 3.5 units. The buoyancy parameter is 0.1.

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